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Finite Capacity Single Server Queuing System with Reverse Reneging in Fuzzy Environment

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Abstract: In this paper, we introduce the fuzzy concept for reverse reneging which helps to find the expected system size (L_S) in terms of crisp value for a finite capacity single server queuing system. By considering the arrival rate, service rate and the reneging rate as trapezoidal fuzzy numbers, we convert all these fuzzy numbers into crisp values by using Robust Ranking Technique. Further we use this technique to find the expected system size. Finally, the analytical results are numerically verified by changing the values for different parameters under crisp environment for L_S.

Keywords: Reverse Reneging, Queuing Systems, Robust Ranking Technique, Fuzzy numbers and Crisp values.

I. INTRODUCTION

Now a day's managing business has become a challenging task due to highly uncertain economic environment. One of the most uncertain aspects of business is customer's behavior. Customers have become more selective. A customer may get impatient due to higher level of expectations, delay in service, facility lack otherwise he decides to leave the facility before completion of service. This behavior leads to the loss in goodwill of the company and it becomes the most important threat to any business. But when it comes to sensitive businesses like investment, selecting a restaurant for dinner, selecting a service station or choosing a saloon, level of impatience of customers depends upon the amount of trust they show with particular firm. Customers are willing to spend more time with high level of patience with the firms having a large consumers' base. For instance, if someone is planning to dine out, he is willing to wait for much longer in order to get access to a well-known restaurant. It is also obvious that well-known brands have a large customers' base. Hence, a large customers' base also works as a trust factor for a customer and the patience level of the customer is high in such cases. This behaviour is referred to as reverse reneging, according to which, higher system size results in high patience and vice-versa. By keeping this in mind, researchers across the world study various stochastic queuing models with reneging. In this model, reneging is a function of system size. When there are more number of customers in the system and vice versa, which is reneging in reverse sense called as **Reverse Reneging**. The premier work on customer impatience in queuing theory appears in Haight [6]. He investigated M/M/1 queue with balking in which there is a greatest queue length at which the arrival will not balk. Haight [7] studies a queue with reneging in which he studies the problem like how to make rational decision while waiting in the queue, the probable effect of this decision etc. Some queuing problems with balking and reneging I was studied by Ancker and Gafarian [1].

Rakesh Kumar and Bhupender Kumar Som [14] developed and introduce the concept of reverse reneging queuing theory. Ancker and Gafarian [2] obtain results for a pure balking system by setting the reneging parameter equal to zero. Armony et al [3] study sensitivity of the optimal capacity to customer impatience. They observe that the prevention of reneging during service can substantially reduce the total cost of lost sales and capacity. Rao [15] study a queuing process of type M/G/1 where units balk as well as renege. For further studies on balking and reneging one may refer Rao [16], Robert [17], Baccelli et al., [4], Gupta [5], Kumar and Sharma [11], Kumar et al., [12] etc.

We have many methods for converting fuzzy into crisp for which Robust Ranking technique is the most convenient method. By using this method we can convert fuzzy into crisp values. A general approach for queuing systems in a fuzzy environment based on zadeh's extension principle, which reduces fuzzy queue into family of crisp values is developed by Kao et al[10]. Fuzzy queues are potentially much more useful and realistic than the commonly used crisp queues. Li and Lee[13] investigated analytical results for two typical fuzzy queues FM/FM/1/∞, M/F/1/∞ where F represents fuzzy time and FM represents fuzzified exponential distribution using zadeh's extension principle.

Many researchers like R.R.yager and S.P Chen[18] have discussed Ranking technique. Julia Rose Mary and Angel Jenitta [8] proposed the cost analysis for bi-level threshold policy and single vacation of an unreliable server with fuzzy parameters. Recently Julia Rose Mary and Pavithra [9] also discussed the analysis of FM/M(a,b)/1/MWV queuing model. With aid of the available literatures we study a finite capacity single server queuing system with reverse reneging which is developed by Rakesh Kumar and Bhupender Kumar Som [14].

Owing to this practically valid aspect, in this paper we develop a fuzzy single server stochastic queuing model with reverse reneging.

II. MODEL ASSUMPTIONS

The queuing model is based on the following assumptions:
1. The arrivals to the queuing system occur one by one in accordance with a Poisson process with mean rate λ. The inter-arrival times are independently, identically and exponentially distributed with parameter λ.
2. There is a single-server and the customers are serviced one by one. The service times are independently, identically and exponentially distributed with parameter μ.
3. The capacity of the system is finite, say N.
4. The customers are served in order of their arrival, i.e. the queue discipline is First-Come, First-Served.
5. In reverse reneging, the impatience (reneging) is more if there is less number of customers in the system and vice-versa. The reneging times are independently, identically, and exponentially distributed with parameter η. Reverse reneging occurs with rate {N−(n−1)}η, where N, n and 1/η are the system capacity, number of customers in the system, and the reneging times, respectively.

With the help of these model descriptions we determine a finite single server queuing system with reverse reneging with fuzzy parameters.

Suppose the arrival rate λ, service rate μ and reneging rate η are approximately known and can be represented as fuzzy set $\bar{\lambda}$, $\bar{\mu}$ *and* $\bar{\eta}$ where,

$\bar{\lambda}$ ={t, θ (t)/ $\theta_{\bar{\lambda}}$ (t)/t∈S($\bar{\lambda}$)}, $\bar{\mu}$ ={x, $\theta_{\bar{\mu}}$ (x)/x∈S($\bar{\mu}$)} and $\bar{\eta}$ ={u, $\theta_{\bar{\eta}}$ (u)/u∈S($\bar{\eta}$)}.

Here $\theta_a(b)$ and S(a) denote the membership function and support of **a** where a= $\bar{\lambda}, \bar{\mu}, \bar{\eta}$ are fuzzy numbers and b= t, x, u are the crisp values corresponding to arrival rate, service rate and reneging rate respectively.

On the basis of the concept of α-cut we develop a mathematical programming approach for deriving the α-cuts for $\bar{\lambda}, \bar{\mu}, \bar{\eta}$ as crisp intervals which are given by

$\bar{\lambda}$ (α)={t∈T/ $\theta_{\bar{\lambda}}$ (t)≥α},

$\bar{\mu}$ (α)={x∈X/ $\theta_{\bar{\mu}}$ (x)≥α},

$\bar{\eta}$ (α)={u∈U/ $\theta_{\bar{\eta}}$ (u)≥α}

where 0<α≤1. Hence a fuzzy queue can be reduced to a family of crisp queues with different α-cuts{λ(α)/0<α≤1},{μ(α)/0<α≤1} and {η(α)/0<α≤1}.

Let the confidence interval of the fuzzy sets λ(α),μ(α) and η(α) be[l_{λ} (α) u_{λ} (α)], [$l_{\mu}(\alpha)$ u_{μ} (α)] and [l_{η} (α) u_{η} (α)] according to the classical finite single server model with reverse reneging, the expected system size (L_s) is given by

$$L_s = \sum_{n=0}^{N} n\left[\prod_{r=1}^{n}\frac{\lambda}{\mu+\{N-(r-1)\}\eta}\right]P_0$$

$$\text{where } P_0 = \left[1+\sum_{n=1}^{N}\left\{\prod_{r=1}^{n}\frac{\lambda}{\mu+\{N-(r-1)\}\eta}\right\}\right]^{-1}$$

By applying the fuzzy variable for arrival rate, service rate and reneging parameter we get,

$$L_s = \sum_{n=0}^{N} n\left[\prod_{r=1}^{n}\frac{t}{x+\{N-(r-1)\}u}\right]P_0$$

$$\text{where } P_0 = \left[1+\sum_{n=1}^{N}\left\{\prod_{r=1}^{n}\frac{t}{x+\{N-(r-1)\}u}\right\}\right]^{-1}$$

III. ROBUST RANKING TECHNIQUE

To find the characteristics of system interest in terms of crisp value we defuzzify the numbers into crisp ones by a fuzzy number using ranking method. By giving a convex fuzzy number 'a', the Robust Ranking index is defined by,

$$R(\bar{a}) = \int_0^1 0.5(a_\alpha^L + a_\alpha^U)d\alpha$$

where $(a_\alpha^L + a_\alpha^U)$ is the α-level cut of the fuzzy number $\bar{a}$. In this chapter we use this method for ranking the fuzzy numbers. The Robust ranking index $R(\bar{a})$ gives the representative value of the fuzzy number $\bar{a}$, which satisfies the linearity and additive property.

IV. NUMERICAL EXAMPLE

To study the effect of parameter on the system characteristics, Numerical evaluation is performed and the various results are displayed with graphs.

Consider a fuzzy finite capacity single server with reverse reneging. The corresponding parameters such as arrival rate, service rate and reverse reneging parameter are fuzzy numbers. By letting

λ = [0.03, 0.04, 0.05, 0.06],
μ = [0.1, 0.15, 0.2, 0.25], and
η = [0.08, 0.1, 0.12, 0.14]

whose intervals of confidence are [0.03+α 0.06-α], [0.1+α 0.25-α] and [0.08+α 0.14-α] respectively. The membership function of the trapezoidal fuzzy number (0.03,0.04,0.05,0.06) is given by

$$\theta(p) = \begin{cases} \dfrac{p-0.03}{0.1}, & 0.03 \le p \le 0.04 \\ 0.03, & 0.04 \le p \le 0.05 \\ \dfrac{0.06-p}{0.1}, & 0.05 \le p \le 0.06 \\ 0, & otherwise \end{cases}$$

The α-cut of the fuzzy numbers (0.03, 0.04, 0.05, 0.06) is (0.03+0.1α, 0.06-0.1α) for which

R($\bar{\lambda}$)=R(0.03,0.04,0.05,0.06)

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$$= \int_0^1 0.5(0.03+0.06) = \int_0^1 0.5(0.09) = 0.05.$$

By substituting the above values the expected system size for fuzzy finite capacity single server with reverse reneging are tabulated.

Table 1: Expected system size for a fuzzy finite capacity of single server with reverse reneging (N=2, η=0.1)

	λ=0.05	λ=0.1	λ=0.15	λ=0.2	λ=0.25
μ=0.35	0.2605	0.446	0.5848	0.6926	0.7789
μ=0.45	0.2215	0.3874	0.5162	0.6399	0.7034
μ=0.55	0.1928	0.3426	0.4624	0.5603	0.6418
μ=0.65	0.1708	0.3071	0.4188	0.5118	0.5903
μ=0.75	0.1533	0.2784	0.3829	0.4711	0.5467

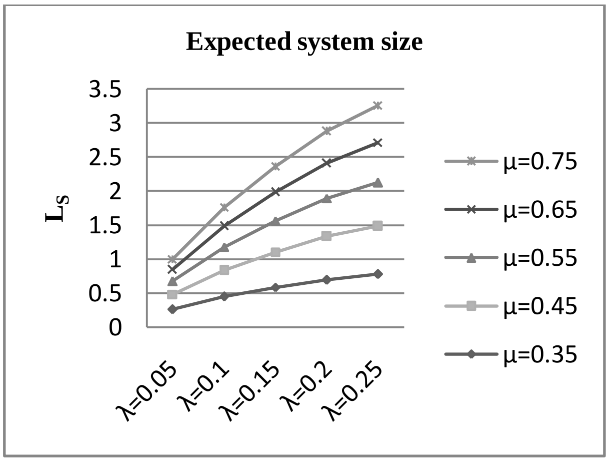

Figure 1. L_s versus (λ, μ)

From the table and figure, we observe that the expected system size increases with increasing arrival rate and decreases with increasing service rate.

By making few changes in the values of fuzzy numbers and proceeding similarly the Robust Ranking Indices for the fuzzy numbers $\bar{\lambda}, \bar{\mu}$ and $\bar{\eta}$ are calculated for $R(\bar{\lambda}) = 0.4,\ R(\bar{\mu}) = 0.05$ and $R(\bar{\eta}) = 0.2$

Table 2 Expected system size for a fuzzy finite capacity single server with reverse reneging (N=2, μ=0.05)

λ / η	0.4	0.5	0.6	0.7	0.8
0.2	1.1719	1.2432	1.2958	1.3361	1.368
0.4	0.9529	1.0412	1.1097	1.1644	1.2091
0.6	0.8013	0.8936	0.9679	1.0303	1.0804
0.8	0.6909	0.7823	0.8578	0.9215	0.9758
1	0.6702	0.6955	0.7701	0.834	0.8894

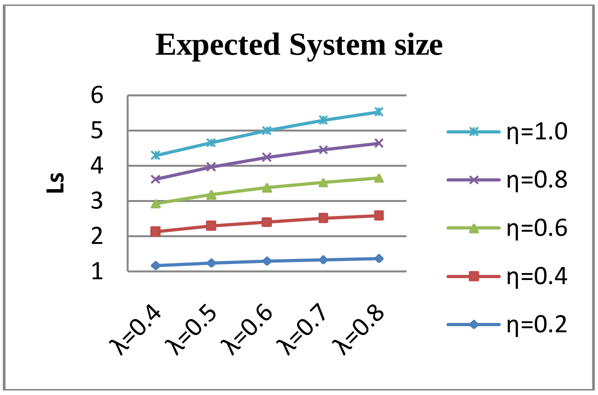

Figure 2. L_s versus (λ, η)

Thus by tabulating the above table values in the figure 2, we find from the table(2) and graph (2) that the expected number of customers in the system increases when the arrival rate increases and also decreases when the reneging rate increases.

V. CONCLUSION

In this paper the fuzzy concept of reverse reneging is introduced. The arrival rate, service rate and reneging rate are fuzzy number. Further the fuzzy problem has been converted into crisp problem by using Robust Ranking Technique. Thus by applying the R.R.T, we find that the expected system size (L_S) increases with respect to arrival rate and decreases for the service rate. Further by making few changes in R.R.T we find that the expected number of customers in the system increases when the arrival rate increases and also decreases when the reneging rate increases. Moreover, the efficient results are also verified graphically.

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